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Dynamic Analysis of a Homogeneous Perforated Rectangular Plate Using Three-Dimensional Elasticity Theory and the Point Radial Basis Function Method (RPIM)

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Abstract

There have been significant research works on the free vibration behavior of plates with holes owing to its importance in engineering applications such as aerospace, civil, marine, and mechanical engineering fields. It is well known that the inclusion of holes in plate structure changes the dynamic behavior of such plates and gives rise to many difficulties in numerical studies especially for three-dimensional cases. In the current paper, the free vibration behavior of homogeneous rectangular plates with a centrally placed circular hole is analyzed based on the three-dimensional elasticity solution along with the meshless technique known as the Radial Point Interpolation Method (RPIM). The approximation scheme of displacement field in terms of RPIM shape functions is made based on multiquadric radial basis functions combined with polynomial basis functions. The derivation of the frequency equation for motion is performed through the principle of stationary energy and the linear elastic governing differential equations. Contrary to several other conventional techniques developed for perforated plates, the present approach does not require any domain decomposition in the neighborhood of the hole or subtracting the hole energy from total strain energy. A MATLAB code is written for the implementation of the proposed formulation. The computed values are verified using finite element method performed by ABAQUS as well as other benchmark solutions presented in literature. A detailed study on the effect of cutout size and boundary conditions on the natural frequency of the plates is carried out. The results clearly indicate the accuracy, robustness, and effectiveness of the proposed meshless three-dimensional formulation.

Keywords: Free vibration, Three-dimensional elasticity, Meshless methods, Radial point interpolation method, Perforated plates, Natural frequencies.

1 | Introduction

Plate structures are important structural elements in a variety of engineering applications, such as aircraft fuselages, ship hulls, bridge decks, pressure vessels, machine elements, and civil infrastructure systems. In

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most cases, the plates used in practice may have cut-outs which are created due to various reasons like reducing the weight of the structure, for easy inspection, for ventilation, or for functional considerations. Despite the advantages of having cut-outs in the plates, their presence affects considerably the stiffness distribution, stress concentration, and dynamics of the plate.

The early studies on the vibration behavior of plates having holes have mainly used the analytical and semianalytical methods. In their investigation on the transverse vibration of a rectangular plate with holes using analytical approach, Laura et al. [1] used an optimized Rayleigh-Ritz model. The dynamic behavior of a rectangular plate with holes using a finite difference model was determined by Aksu and Ali [2]. Rajamani and Prabhakaran [3] considered a composite plate with holes using equivalent loading technique. Lam et al. [4] used a modified Rayleigh-Ritz technique with domain discretization to evaluate the vibration behavior of plates with holes, while Liew et al. [5] applied discrete Ritz method for thin plates with central holes. Using an energy subtraction method, Takahashi [6] subtracted the energy of the hole region from the overall energy of the plate. Kwak and Han [7] adopted a new coordinate coupling scheme for plates with holes using two different coordinate systems for the plate and hole regions.

Despite their efficiency, most of these classical methods are plagued with several shortcomings. The analytical solutions are limited to ideal geometry and boundary conditions, while the semi-analytical solutions involve very tedious domain discretization steps and formulation processes. In addition, most works use classic or shear deformation plate theory, which might fail to model the 3D stress state of moderately thick and thick plates.

The application of the 3D elasticity theory provides a sound basis for vibration analysis since it considers all the stresses and displacements without making any kinematic assumptions like in case of equivalent plate theories. Several investigations have established that 3D solutions perform better especially for thick and geometrically complex problems [8]. But when implementing the 3D elasticity solution in numerical schemes using conventional mesh dependent methods, some difficulties arise.

Various meshless techniques have recently been recognized as promising solutions compared to finite element schemes owing to their ability to deal with complicated geometry, moving boundaries, evolving discontinuities, and node distributions. Within this family of methods, Radial Point Interpolation Method (RPIM) proposed by Liu et al. [9] has received much interest owing to its Kronecker delta property, easy imposition of essential boundary conditions, and good rate of convergence. Over the years, RPIM scheme has found many applications in solving various types of problems like elasticity, fracture mechanics, piezoelectric materials, wave propagation, vibration problems etc.

Recent research efforts have indicated the applicability of RPIM based methods for dynamic problems. For example, Xia and Long [8] formulated the static and free vibration problems for moderately thick functionally graded plates using the local RPIM method. Sun et al. [10] developed a Modified Radial Point Interpolation Method (M-RPIM) approach for analyzing free vibrations of solids. In another study, He et al. [11] used meshless techniques for analyzing the bending and free vibrations of ribbed plates with holes.

However, researches focusing on the three-dimensional vibration analysis of perforated plates with the meshfree technique are few. Simplifications in form or domain decomposition in relation to cutout are some examples that can be observed in existing formulations. In light of this, a new efficient three-dimensional analysis is required, which does not introduce extra geometry into the perforated plate.

Guided by this discussion, the current paper explores the natural vibrations of rectangular plates with a centrally placed circular cutout using three-dimensional RPIM formulation. Equations of motion are developed based on the principles of minimum energy of the system and linear elasticity theory, thereby producing an ordinary eigenvalue problem. The present scheme excludes the use of cutout energy subtraction and division of the domain, which are traditional ways of handling such problems. Computations conducted in the current work are compared to results from the finite element method using software ABAQUS and also those available in the literature. Effects of cutout diameter and plate boundary condition on natural frequencies are further explored.

2 | Literature Review

2.1 | Classical Studies on Free Vibration of Plates with Cutouts

The vibration analysis of plates with perforations has been one of the areas of significant academic interest for a considerable number of years due to the extensive use of openings in engineering structures. In early studies, both analytic and semianalytic methods were used. Laura et al. [1] used an optimal form of the Rayleigh-Ritz procedure for studying the dynamic behavior of transversely vibrating simply supported rectangular plates with rectangular cutouts. Their analysis showed that cutouts have a very strong effect on the frequency spectrum of plate structures.

In a later study, Aksu and Ali [2] presented a finite difference method to analyze the dynamic behavior of rectangular plates with openings. They modeled the opening problem using external loads and studied the dynamic behavior of composite plates with openings. Even though their technique simplified the problem because of the difficulty caused by geometry, it was applicable only to certain configurations.

The computational problem can be solved through the development of a modified Rayleigh-Ritz approach suggested by Lam et al. [4], where the plate with holes was partitioned into different subdomains. Likewise, Liew et al. [5] developed a discrete Ritz method to study thin isotropic plates with cutouts at their center positions. Takahashi [6] developed an energy subtraction approach based on subtracting the strain energy of the cutout section from the total strain energy of the complete plate. But, such a process frequently leads to complicated mathematical formulations.

This problem could be addressed by Kwak and Han [7], who enhanced classical methods through the development of an independent coordinate coupling technique utilizing separate coordinates for the plate and cutout sections. Such approaches worked well only for simpler geometries.

2.2 | Meshless Methods and the Development of Radial Point Interpolation Method

The shortcomings of the numerical schemes that were based on meshes led to the emergence of mesh-free schemes. Mesh-free schemes are unlike finite element schemes since in such schemes element connections are not required, hence there is no need for any meshes and it makes it easier to analyze complicated geometries.

Some of the mesh-free schemes include the RPIM which was invented by Liu and Gu [12]. The RPIM scheme employs radial basis functions along with polynomial terms in order to construct the interpolation functions and hence shape functions that satisfy the Kronecker delta function.

It was also shown that RPIM is applicable to simulations of piezoelectric media and that it possesses excellent convergence properties over some other meshless methods [13]. The freedom related to arbitrary node placement and local supports makes RPIM a powerful tool for the solution of elastic, fracture mechanics, wave propagation, and dynamic structural problems.

2.3 | Radial Point Interpolation Method Applications in Vibration Analysis

The use of RPIM in vibration problems has been extended extensively over the last two decades. Xia and Long [8] formulated the meshless local RPIM approach to investigate the static behavior and free vibration of moderately thick heterogeneous plates. The investigation proved that RPIM could deliver good accuracy when compared with finite element results without having any need for extensive meshing. Sun et al. [10] employed the M-RPIM to analyze the free vibrations of solids. Fantuzzi et al. [14] used the conforming RPIM in the static and free vibration analysis of laminated composite plates. The smoothed Hermite RPIM was developed by Cui et al. [15] for thin plate analysis. Liew et al. [16] formulated the mesh-free RPIM to perform buckling and vibration analysis of plates. It was concluded that RPIM can predict frequencies accurately even in the case of structures possessing geometric discontinuity and material inhomogeneity.

2.4 | Recent Developments in Plate Vibration Analysis (2021–2025)

Several advancements have been achieved in vibration analysis of plate structures within the recent years. Devarajan and Kapania [17] have used an isogeometric level set method for studying the behavior of thermal buckling of curved stiffened composite plates with irregular holes.

Nonlinear free vibration of bidirectional functionally graded material truncated conical shells has been studied by Shadmani et al. [18]. The vibration behaviors of LFGMs plates with openings were explored by Yang et al. [19] applying the spectral shift Legendre method. The authors also performed an investigation of free vibration behaviors of three-dimensional solids with holes utilizing the Ritz method and three-dimensional elasticity theory [20].

Vaghefi et al. [21] have developed a new meshless method of Tchebychev-radial point interpolation for analyzing the post-buckling behavior of three-dimensional functionally graded plates.

Bending and free vibration analysis of a ribbed plate with a circular hole was made using the FSDT meshless technique by He et al. [11]. Computational investigation of the non-linear dynamic characteristics of a perforated plate was made by VeisiAra et al. [22]. These achievements indicate a rising level of interest in numerical methods of analysis of complicated plates.

2.5 | Research Gap

Despite all the advancements made in the areas of perforated plate and meshless analyses, a number of shortcomings are apparent.

In the first place, the vast majority of studies conducted on perforated plates utilize the classical theories of plates or semi-analytical methods requiring domain decomposition and energy subtraction methods among other things. Secondly, very few works have used fully three-dimensional elasticity theories to study vibrations in perforated plates. Thirdly, the utilization of RPIM for perforated plates, particularly combined with fully three-dimensional elasticity theories, is still rare.

Thus, a need arises for a computational technique that integrates the precision of three-dimensional elasticity along with the adaptability of RPIM but without getting into the cumbersome numerical difficulties of existing perforated plates techniques.

This research aims to rectify such issues by proposing a three-dimensional RPIM formulation for studying the free vibrations of homogeneous rectangular plates with a circular hole at their center. In doing so, it achieves a solution without resorting to any energy extraction method and avoids the requirement of dividing the structure into subdomains while remaining accurate through comparison with ABAQUS and available results.

3 | Element-Free Method

To interpolate a field function such as displacement $u(x)$ at a point x_Q or $u(x, x_Q)$, within the domain Ω—whose boundary and interior are defined using an arbitrary distribution of N nodes at positions x_i $(i = 1,2,\cdots,N)$—a region called the support domain is considered, centered at x_Q with an arbitrary shape and size. This region is defined such that a certain number of the distributed nodes lie inside it. In this method, to estimate the desired field function at point x_Q, only the nodes located within its support domain are used, and all other nodes have no effect on the value of the field function at x_Q.

The pointwise radial basis interpolation method, using the nodes inside the support domain of x_Q along with radial basis and polynomial functions, interpolates the value $u(x, x_Q)$ as follows [12].

$$u^h(x, x_Q) = \sum_{i=1}^{n} R_i(x)a_i + \sum_{j=1}^{m} P_j(x)b_j = R^T(x)a + P^T(x)b. \quad \textbf{(1)}$$

Such that $R_i(x)$ and $P_j(x)$ denote the radial basis functions and polynomial basis functions, respectively; n is the number of nodes within the support domain of x_Q; m is the number of polynomial basis terms; and a_i and b_j are coefficients that will be determined. And:

$$R^T(x) = [R_1(x), R_2(x), \cdots, R_n(x)]. \quad \textbf{(2)}$$

$$P^T = [P_1(x), P_2(x), P_3(x), \ldots, P_m(x)]. \quad \textbf{(3)}$$

$$a = [a_1, a_2, a_3, \ldots, a_n]^T. \quad \textbf{(4)}$$

$$b = [b_1, b_2, b_3, \ldots, b_m]^T. \quad \textbf{(5)}$$

There are several types of radial basis functions, among which multiquadratic and Gaussian functions have been widely used. In the present work, the multiquadratic type has been employed.

$$R_i(x, y, z) = [{r_i}^2 + (\alpha_c d_c)^2]^q. \quad \textbf{(6)}$$

Such that α_c and q are the parameters of the radial basis function, and d_c is the average nodal spacing.

The radial basis function depends on the distance between the point x_Q and the node x_i. The polynomial basis function for the three-dimensional case with $m = 4$ is shown.

$$P^T(x) = [1 \quad x \quad y \quad z]. \quad \textbf{(7)}$$

The coefficients a_i and b_j in *Eq. (1)* for the point x_Q are determined by enforcing this equation at the n nodes within its support domain, as follows.

$$\sum_{i=1}^{n} R_i(x_k)a_i + \sum_{j=1}^{m} P_j(x_k)b_j = u_k = u(x_k) \quad (k = 1,2, \cdots, n) \quad \textbf{(8)}$$

is the value of the function $u$at the k-th node. The above equation can be expressed in matrix form as follows.

$$R_Q a + P_m b = \hat{u}. \quad \textbf{(9)}$$

$$\hat{u} = [u_1, u_2, u_3, \ldots, u_n]^T. \quad \textbf{(10)}$$

$$R_Q = \begin{bmatrix} R_1(x_1, y_1, z_1) & R_2(x_1, y_1, z_1) & \ldots & R_n(x_1, y_1, z_1) \\ R_1(x_2, y_2, z_2) & R_2(x_2, y_2, z_2) & \ldots & R_n(x_2, y_2, z_2) \\ \vdots & \vdots & \vdots & \vdots \\ R_1(x_n, y_n, z_n) & R_2(x_n, y_n, z_n) & \ldots & R_n(x_n, y_n, z_n) \end{bmatrix}_{n\times n}. \quad \textbf{(11)}$$

$$P_m = \begin{bmatrix} P_1(x_1, y_1, z_1) & P_2(x_1, y_1, z_1) & \ldots & P_m(x_1, y_1, z_1) \\ P_1(x_2, y_2, z_2) & P_2(x_2, y_2, z_2) & \ldots & P_m(x_2, y_2, z_2) \\ \vdots & \vdots & \vdots & \vdots \\ P_1(x_n, y_n, z_n) & P_2(x_n, y_n, z_n) & \ldots & P_m(x_n, y_n, z_n) \end{bmatrix}_{n\times m}. \quad \textbf{(12)}$$

The above system consists of $n$equations and $n + m$ unknowns. To solve it, the following $m$additional constraint equations are introduced [9].

$$P_m^T a = 0. \quad \textbf{(13)}$$

Using *Eqs. (9)* and *(13)*, we obtain [13]:

$$a = S_a\hat{u}, b = S_b\hat{u}, \quad \textbf{(14)}$$

where

$$S_a = R_Q^{-1}[I - P_m S_b]. \quad (15)$$

$$S_b = \left[P_m^T R_Q^{-1} P_m\right]^{-1} P_m^T R_Q^{-1}. \quad (16)$$

By substituting a and b into *Eq. (1)*, the interpolation (shape) function Ψ is obtained as:

$$u^h(x, x_Q) = [R^T(x)S_a + P^T(x)S_b]\hat{u} = \Psi\hat{u}, \quad (17)$$

$$\Psi(x) = R^T(x)S_a + P^T(x)S_b = [\phi_1 \; \phi_2 \; \cdots \; \phi_n], \quad (18)$$

where ϕ_i is the shape function corresponding to the i-th node. The derivative of the interpolation function with respect to the independent variables $l = x, y, z$ is given by:

$$\Psi_{,l} = [R_{1,l} \quad R_{2,l} \quad \cdots \quad R_{n,l}]S_a + [0 \quad x_{,l} \quad y_{,l}]S_b, \quad (19)$$

where the subscript (Ψ, l) denotes differentiation with respect to l.

Fig. 1 shows a general schematic of a perforated rectangular plate and a cubic support domain for a point x_Q. In the figure, the nodes around the hole are shown only on the upper and lower surfaces of the plate.

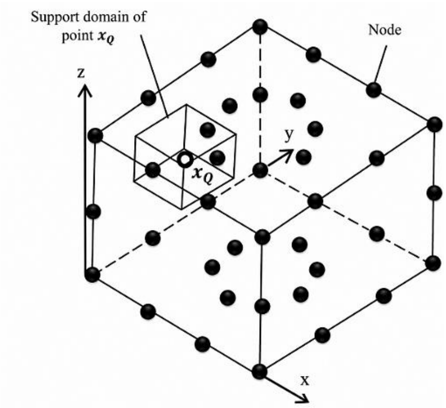

Fig. 1. Lattice points forming the planar geometry and the cubic protective zone/point region.

4 | Governing Equations

Consider a rectangular plate with length a, width b, and height h. A Cartesian coordinate system (x, y, z) is used to describe the geometry and dimensions of the plate, as well as the small displacements of the elastic plate (*Fig. 2*).

The displacement field corresponding to this coordinate system is given by:

$$u = [u \; v \; w]^T, \quad (20)$$

where u, v, and w represent the displacements in the x, y, and z directions, respectively.

The stress–strain relationship is expressed as:

$$\sigma = D\,\varepsilon. \quad (21)$$

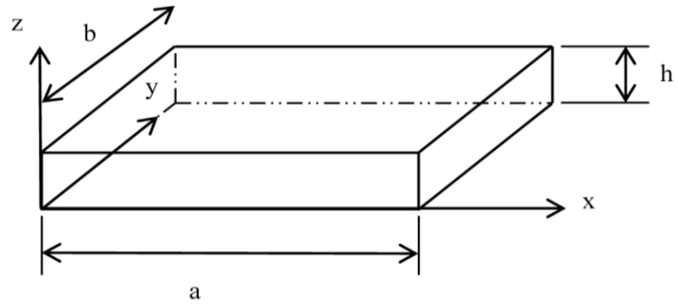

Fig. 2. Dimensions of the rectangular plane and the utilized coordinate system.

According to three-dimensional elasticity theory, we have:

$$\sigma = [\sigma_x\ \sigma_y\ \sigma_z\ \tau_{yz}\ \tau_{xz}\ \tau_{xy}]^T, \tag{22}$$

$$\varepsilon = [\varepsilon_x\ \varepsilon_y\ \varepsilon_z\ \gamma_{yz}\ \gamma_{xz}\ \gamma_{xy}]^T, \tag{23}$$

and D is the elastic matrix, which for homogeneous materials is given as:

$$D = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}-\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}-\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}-\nu \end{bmatrix}. \tag{24}$$

In the above relation, E is the Young's modulus and ν is Poisson's ratio. The relationship between strain and displacement is given as:

$$\varepsilon = Lu, \tag{25}$$

where the differential operator matrix L is defined as

$$L = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix}. \tag{26}$$

Using the energy principle for free vibration analysis of a non-damped plate, we have:

$$\frac{d}{dt}(\Pi + T) = 0, \tag{27}$$

where the linear elastic strain energy Π and kinetic energy T for a rectangular plate are expressed in integral form as:

$$\Pi = \frac{1}{2}\int_0^a \int_0^b \int_0^h \varepsilon^T \sigma \, dz \, dy \, dx, \tag{28}$$

$$T = \frac{1}{2}\int_0^a \int_0^b \int_0^h \rho \, \dot{u}^T \dot{u} \, dz \, dy \, dx, \tag{29}$$

where $\rho(x, y, z)$ is the plate density.

The displacement field of the plate is assumed as:

$$u(x, y, z, t) = \begin{bmatrix} u(x, y, z, t) \\ v(x, y, z, t) \\ w(x, y, z, t) \end{bmatrix} = \begin{bmatrix} U(x, y, z) \\ V(x, y, z) \\ W(x, y, z) \end{bmatrix} e^{i\omega t} = \bar{U} e^{i\omega t} \tag{30}$$

where ω is the circular frequency and $\bar{U}$ is the displacement amplitude vector of the plate.

Now, assuming n nodes within the support domain of a given point in the plate, the displacement amplitudes are approximated as

$$\bar{U} = N\bar{\bar{U}}, \tag{31}$$

where N is the shape function matrix:

$$N = \begin{bmatrix} \phi_1 & 0 & 0 & \phi_2 & 0 & 0 & \cdots & \phi_n & 0 & 0 \\ 0 & \phi_1 & 0 & 0 & \phi_2 & 0 & \cdots & 0 & \phi_n & 0 \\ 0 & 0 & \phi_1 & 0 & 0 & \phi_2 & \cdots & 0 & 0 & \phi_n \end{bmatrix}. \tag{32}$$

$$\bar{\bar{U}} = [u_1 \; v_1 \; w_1 \; \cdots \; u_n \; v_n \; w_n]^T. \tag{33}$$

By defining the strain–displacement matrix operator B, the strains, stresses, and displacements can be written as:

$$\varepsilon(x, y, z, t) = B\bar{\bar{U}} e^{i\omega t}. \tag{34}$$

$$u(x, y, z, t) = N\bar{\bar{U}} e^{i\omega t}. \tag{35}$$

$$\sigma(x, y, z, t) = DB\bar{\bar{U}} e^{i\omega t}. \tag{36}$$

Substituting *Eqs. (34)– (36)* into *Eqs. (28)* and *(29)* and applying *Eq. (27)*, and through a discretization (assembling) process over the support domains of all selected sampling points within the plate, a standard eigenvalue problem is obtained as:

$$(K - M\omega^2)\bar{\bar{U}} = 0, \tag{37}$$

Where

$$M = \int_0^a \int_0^b \int_0^h \rho \, N^T N \, dz \, dy \, dx. \tag{38}$$

$$K = \int_0^a \int_0^b \int_0^h B^T DB \, dz \, dy \, dx. \quad (39)$$

As shown, in the present method, there is no need for techniques traditionally used in the analysis of perforated plates, such as subtracting the energy contribution of the hole region from the total energy [6], or decomposing the region around the hole into several subdomains [4].

5 | Numerical Results

In the present study, free vibrations of a rectangular plate with length 10, width 5, and thickness 0.5 m are investigated. The density of the plate is taken as $\rho = 1\ kg/m^3$, the Young's modulus as $E = 30\ MPa$, and Poisson's ratio as $\nu = 0.3$.

Each boundary condition is represented by four Latin letters, each corresponding to one face of the relatively thick plate. The first letter from the left indicates the boundary condition of face 1, the second letter corresponds to face 2, and so on. For example, in the SCSC boundary condition, faces 1 and 3 are simply supported, while faces 2 and 4 are clamped. The letter F represents a free boundary condition. The faces of the plate subjected to different boundary conditions are shown in *Fig. 3*.

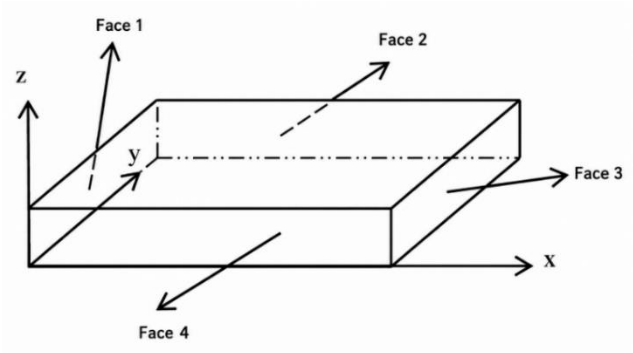

Fig. 3. Faces of the plate under different boundary conditions.

To determine the natural frequencies using the element-free method, a MATLAB code was developed. In addition, the plate was simulated using ABAQUS.

In this work, a two-point Gauss integration scheme is used. In the process of deriving the mass and stiffness matrices, the relatively thick rectangular plate is divided into background cells with a cubic structure. Gauss points for each background cell are defined in all three directions x, y, and z, and their support domains are computed. A cubic support domain is used in this study. *Fig. 4* shows the generated background cells and Gauss points for a single cell.

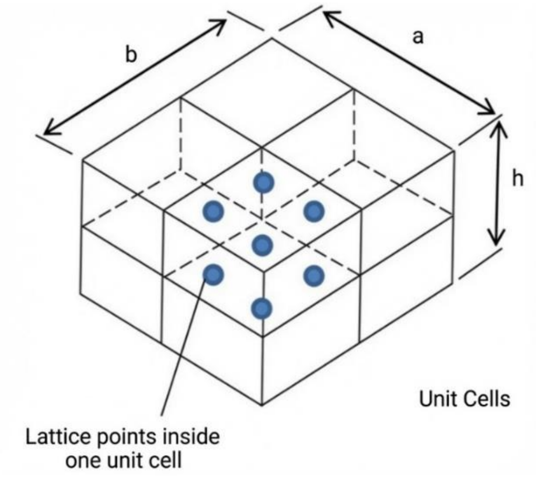

Fig. 4. Background cells and Gauss points for a single cell.

The results obtained from the developed code are presented in terms of convergence, along with reference results [8] and ABAQUS results, for a homogeneous plate without a hole and with fully clamped boundary conditions. These results are provided in *Table 1*.

Table 1. Convergence and comparison of the fundamental dimensionless frequency of a homogeneous plate without a hole and with CCCC boundary condition.

CCCC	Number of Nodes N_x	N_y	N_z	Method
93,678	1	9	3	
91,998	19	10	3	present
91,828	21	11	3	
91,617	23	12	3	
90,895				ABAQUS
91,482				Ref. [8]

5.1 | Homogeneous Perforated Plate

To investigate the effect of a hole on the natural frequencies of a homogeneous rectangular plate, a circular hole is placed at the center of the plate. The number of nodes in the x, y, and $z$directions is 21, 11, and 3, respectively. The convergence of the results is studied by varying the number of nodes around the hole. The nodes around the hole are distributed at equal intervals. *Table 2* presents the results and their convergence, along with ABAQUS results, for $r = 0.5$ and the CCCC boundary condition.

Table 3 presents the fundamental frequencies of the perforated plate with $r = 0.5$ for six different boundary conditions, and the results are compared with those obtained from ABAQUS. *Fig. 5* shows the dimensionless fundamental frequencies of the perforated plate with fully clamped boundaries. As can be seen, for the fully clamped plate, the fundamental frequency first decreases and then increases with increasing hole radius.

Table 2. Convergence and comparison of the dimensionless fundamental frequency of the perforated plate with $r = 0.5$ and CCCC boundary condition.

ω_1	Number of Nodes around Hole	Methods
92,711	4	
91,055	5	
92,107	6	
91,850	7	Present
91,821	8	
91,286	9	
90,951	10	
91,373		ABAQUS

Table 3. Comparison of dimensionless fundamental frequencies of the perforated plate with $r = 0.5$ and different boundary conditions.

CFFF	SFSF	SSSF	SSSS	SCSC	CCCC	Method	r(m)
3,362	9,325	15,593	47,474	89,113	91,828	RPIM	0
3,318	8,869	15,423	46,039	89,366	92,107	RPIM	0.5
3,393	9,363	15,702	47,830	88,363	90,895	ABAQUS	0
3,377	9,219	15,716	46,863	88,576	91,373	ABAQUS	0.5

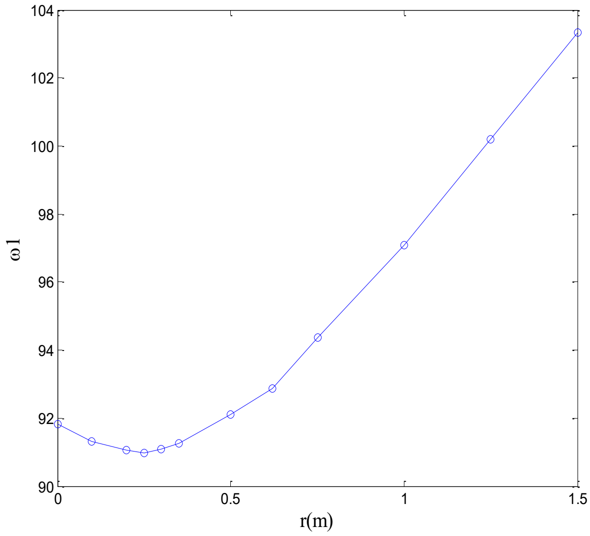

Fig. 5. Effect of hole radius on the dimensionless frequency of a homogeneous rectangular plate with CCCC boundary condition.

6 | Conclusion

This study successfully developed a three-dimensional meshless formulation based on the RPIM integrated with the full theory of linear elasticity for the free vibration analysis of homogeneous rectangular plates with a central circular cutout. The proposed approach effectively derives the governing frequency equation as a standard eigenvalue problem using the principle of stationary potential energy and pointwise radial shape functions.

One of the primary contributions of this work is the elimination of conventional and often cumbersome techniques required in most previous studies on perforated plates, such as strain energy subtraction of the cutout region or multi-domain decomposition around the hole. This simplification enhances computational efficiency while preserving numerical accuracy.

The developed MATLAB implementation was rigorously validated against both benchmark solutions [8] and independent ABAQUS finite element simulations for intact and perforated plates under various boundary conditions. The excellent agreement between the present results and reference data confirms the accuracy, convergence, and robustness of the proposed three-dimensional RPIM formulation.

Parametric studies revealed that the presence of a central cutout significantly affects the natural frequencies, with the fundamental frequency of fully clamped plates exhibiting a non-monotonic behavior (initial decrease followed by an increase) as the hole radius grows. This highlights the complex interplay between mass reduction and local stiffness loss.

In conclusion, the present work provides an efficient and accurate three-dimensional meshless framework for the dynamic analysis of perforated plates. The proposed methodology offers a promising alternative to traditional mesh-based methods, particularly for problems involving geometric discontinuities. Future extensions of this formulation to functionally graded materials, laminated composites, and plates with irregular cutouts are recommended.

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