International Journal of Researches on Civil Engineering with Artificial Intelligence

www.ceai.reapress.com

Int. J. Res. Civ. Eng. AI. Vol. 2, No. 3 (2025) 127–138.

Paper Type: Original Article

Fluid–Structure Interaction in Thin-Walled Cylindrical Shells: A Dynamic Analysis

Hossein Masoomi*

Department of Civil Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran; h.masoomi170@yahoo.com.

Citation:

Received: 11 October 2024 *Revised: 26 January 2025* *Accepted: 13 April 2025*	Masoomi, H. (2025). Fluid–structure interaction in thin-walled cylindrical shells: A dynamic analysis. *International journal of researches on civil engineering with artificial intelligence, 2*(3), 127-138.

Abstract

This study investigates the free vibration characteristics of a partially liquid-filled thin-walled cylindrical shell subjected to impulsive excitation. Liquid storage tanks are extensively used in industrial facilities, nuclear power plants, petrochemical industries, and water distribution systems, where their dynamic behavior under external disturbances plays a critical role in ensuring structural safety and operational reliability. The interaction between the elastic shell and the contained liquid significantly alters the dynamic characteristics of the coupled system, particularly the natural frequencies and vibration modes. In the present work, a vertical circular cylindrical shell with simply supported boundary conditions at its base and a free upper edge is considered. The contained liquid is assumed to be incompressible, inviscid, and Newtonian. The Rayleigh–Ritz method combined with Lagrange's equations is employed to derive the governing equations of motion of the coupled fluid–structure system. The hydrodynamic effect of the liquid is incorporated through the concept of added mass obtained from the solution of the Laplace equation for the fluid velocity potential. Subsequently, the free vibration problem is formulated as a generalized eigenvalue problem. Parametric investigations are conducted to examine the influence of the liquid height ratio and shell geometric characteristics on the natural frequencies of the first and second vibration modes. Numerical results indicate that decreasing the liquid filling height leads to an increase in the natural frequencies of the coupled system. Furthermore, increasing the shell aspect ratio considerably modifies the vibration response. The proposed analytical formulation provides an efficient framework for evaluating the dynamic characteristics of partially filled storage tanks and can be used as a benchmark for validating numerical simulations and supporting the design of fluid-containing shell structures.

Keywords: Fluid–structure interaction, Thin-walled cylindrical shell, Partially filled storage tank, Free vibration analysis, Rayleigh–ritz method, Added mass effect.

1 | Introduction

Liquid storage tanks are widely used in nuclear power plants, petrochemical industries, offshore structures, water supply systems, and aerospace applications. The dynamic behavior of these structures is of considerable importance because external excitations such as earthquakes, impact loads, machinery-induced vibrations, and environmental disturbances may induce significant Fluid–Structure Interaction (FSI) effects. The presence of

Corresponding Author: h.masoomi170@yahoo.com
https://doi.org/10.48314/ijrceai.v2i3.50

the contained liquid alters the dynamic characteristics of the supporting structure through hydrodynamic pressure and added-mass effects, resulting in changes in the natural frequencies, mode shapes, and overall vibration response of the coupled system [1–3].

Thin-walled cylindrical shells constitute one of the most common structural forms used for liquid storage because of their high strength-to-weight ratio and manufacturing efficiency. However, their relatively low structural stiffness makes them sensitive to dynamic loading conditions. In partially filled tanks, the existence of a free liquid surface further complicates the problem due to sloshing phenomena and the associated coupling between the shell deformation and liquid motion [4], [5]. Accurate prediction of the free vibration characteristics of such systems is therefore essential for the safe design and operation of fluid-containing structures.

Early investigations of liquid-filled cylindrical tanks primarily focused on simplified analytical models based on the concept of added mass. Amabili and Paı¨doussis [6] proposed one of the earliest strain-energy formulations for thin cylindrical shells, while Ricciardi [7] developed analytical expressions describing the dynamic behavior of fluid–tank systems. Païdoussis et al. [8] subsequently employed the Rayleigh–Ritz approach to investigate the free vibration characteristics of partially filled storage tanks and demonstrated the importance of fluid height on the natural frequencies of the coupled system.

With advances in computational techniques, researchers have developed increasingly sophisticated approaches for modeling FSI problems. Finite element formulations incorporating acoustic fluid elements, boundary element methods, spectral approaches, and semi-analytical techniques have been widely adopted to account for the coupling effects between shell structures and internal fluids [9–11]. Recent studies have highlighted that neglecting FSI may lead to substantial errors in estimating the dynamic response of liquid-containing systems.

In recent years, particular attention has been devoted to partially filled cylindrical shells because of their practical relevance in industrial applications. Fang et al. [12] proposed a semi-analytical framework for nonlinear vibration analysis of submerged and fluid-filled cylindrical shells and demonstrated the strong influence of fluid coupling on the vibration response. Similarly, researchers have investigated the free vibration behavior of cylindrical shells subjected to both internal and external fluid interaction, showing that the presence of free-surface motion significantly modifies the modal characteristics of the structure [13].

Coupled vibration analyses considering internal structural components and sloshing effects have also been reported. For instance, recent investigations on partially liquid-filled cylindrical shells equipped with internal horizontal plates revealed that liquid filling ratios and internal configurations substantially affect both free and forced vibration characteristics [14]. Furthermore, vibroacoustic studies have shown that the interaction between shell structures and internal fluids may influence sound radiation and acoustic transmission properties, thereby extending the importance of FSI analyses beyond conventional structural design [15].

Despite these advances, many available studies rely on computationally intensive numerical procedures or focus on complex shell configurations and nonlinear behaviors. Comparatively fewer investigations provide efficient analytical or semi-analytical formulations suitable for evaluating the fundamental vibration characteristics of partially filled thin-walled cylindrical tanks while retaining the essential physics of FSI. Such formulations remain valuable for preliminary design, parametric studies, verification of numerical simulations, and engineering applications requiring rapid assessment of dynamic properties.

Therefore, the present study investigates the free vibration characteristics of a vertical thin-walled cylindrical shell partially filled with liquid by employing the Rayleigh–Ritz method in conjunction with Lagrange's equations. The contained fluid is assumed to be incompressible, inviscid, and Newtonian. The hydrodynamic influence of the liquid is incorporated through an added-mass formulation derived from the fluid velocity potential satisfying Laplace's equation. The governing equations of the coupled fluid–structure system are subsequently formulated as a generalized eigenvalue problem.

The effects of liquid filling ratio and shell geometric parameters on the natural frequencies of the first and second vibration modes are systematically examined. The obtained results contribute to a better understanding of the dynamic behavior of partially filled storage tanks and provide an efficient analytical benchmark for future investigations involving more sophisticated FSI models.

The main contributions of this study can be summarized as follows:

I. Development of a semi-analytical formulation for free vibration analysis of partially liquid-filled thin cylindrical shells based on Rayleigh–Ritz and Lagrangian approaches.

II. Incorporation of FSI effects through an analytical added-mass representation derived from velocity potential theory.

III. Investigation of the influence of liquid height ratio and shell geometric characteristics on the natural frequencies of the coupled system.

IV. Provision of benchmark results that may be used for validation of numerical and finite element models of fluid-filled storage tanks.

2 | Literature Review

The dynamic analysis of fluid-containing cylindrical shells has attracted considerable attention over the past several decades because of its importance in engineering applications such as storage tanks, offshore structures, pipelines, nuclear facilities, and aerospace systems. The interaction between the shell structure and the contained fluid modifies the inertial characteristics of the system and significantly influences its vibration response.

The earliest analytical investigations focused on the formulation of shell strain energy and the incorporation of hydrodynamic effects through simplified added-mass concepts. Bleich and DiMaggio [1] established a strain-energy formulation for thin cylindrical shells that became the basis for numerous subsequent studies. Yang [2] extended this framework by examining the dynamic behavior of fluid–tank systems and demonstrating the importance of fluid coupling in predicting structural frequencies. Tang and Chang [3] later applied the Rayleigh–Ritz method to partially filled storage tanks and showed that the liquid filling ratio substantially affects the natural frequencies of the coupled system.

As computational capabilities improved, researchers increasingly adopted numerical approaches to study FSI problems. Finite element formulations coupled with acoustic fluid models enabled more accurate representation of hydrodynamic pressures and shell deformations [4], [5]. Boundary element techniques and hybrid finite element–boundary element methods were also introduced to improve computational efficiency while preserving solution accuracy [6].

Several investigations examined the influence of free-surface motion on the dynamic behavior of liquid-filled shells. These studies demonstrated that sloshing phenomena contribute additional flexibility to the coupled system and may alter both natural frequencies and vibration modes [7], [8]. Neglecting free-surface effects was shown to introduce non-negligible errors, particularly in partially filled configurations.

In recent years, increasing attention has been devoted to partially liquid-filled cylindrical shells due to their widespread industrial applications. Chen et al. [9] developed a semi-analytical formulation for the coupled vibration analysis of partially liquid-filled cylindrical shells equipped with internal horizontal plates. Their results revealed that both the liquid filling ratio and internal structural components significantly influence the free and forced vibration characteristics of the system. The study further demonstrated that linear sloshing effects should be incorporated to achieve realistic predictions of the dynamic response.

Montes et al. [10] proposed an analytical–numerical model to investigate free vibrations of cylindrical shells subjected simultaneously to internal and external fluid interaction. By incorporating the motion of the internal free surface and employing Chebyshev polynomial expansions, they demonstrated that external fluid loading and internal sloshing may considerably modify the modal properties of large shell structures. The same research group later extended their work to nonlinear dynamic analyses of submerged and fluid-filled cylindrical shells [11]. Their semi-analytical approach, based on Sanders–Koiter shell theory and the Rayleigh–Ritz method, successfully predicted both free and forced nonlinear responses of coupled shell–fluid systems.

Recent developments have also focused on vibroacoustic aspects of fluid-filled shells. Fang et al. [12] investigated the vibroacoustic behavior of fluid-filled baffled cylindrical shells submerged in shallow-water environments and reported that fluid coupling influences not only structural vibrations but also acoustic radiation characteristics. These findings broaden the engineering relevance of FSI analyses beyond conventional structural safety assessments.

Additional studies have addressed the effects of shell geometry, boundary conditions, material gradation, and internal components on vibration behavior. Investigations involving functionally graded shells, concentric shell systems, and shells conveying fluids consistently reported that fluid–structure coupling reduces natural frequencies and changes modal distributions [13–15]. Furthermore, the sensitivity of the coupled response to filling levels and aspect ratios has been repeatedly emphasized in the literature.

Despite the substantial progress achieved in this field, several limitations remain. Many recent studies rely on sophisticated finite element simulations or nonlinear formulations requiring considerable computational effort. Although such approaches provide highly accurate predictions, they are often unsuitable for rapid parametric analyses or preliminary engineering design. Moreover, comparatively fewer studies have focused on developing efficient analytical formulations capable of accurately predicting the free vibration characteristics of partially filled thin-walled cylindrical shells while retaining the essential effects of FSI.

Therefore, the present study adopts the Rayleigh–Ritz method in conjunction with Lagrange's equations to establish a computationally efficient analytical framework for the free vibration analysis of partially liquid-filled cylindrical shells. The hydrodynamic contribution of the fluid is incorporated through an added-mass formulation derived from velocity potential theory. This approach enables systematic investigation of the effects of liquid filling ratio and shell geometry on the natural frequencies of the coupled system while maintaining relatively low computational cost.

2.1 | Research Gap and Novelty

Although extensive investigations have been conducted on FSI in cylindrical shells, several gaps can still be identified in the literature; 1) many recent studies employ computationally intensive finite element formulations and nonlinear models that limit their applicability in routine engineering analyses, 2) analytical investigations addressing partially liquid-filled thin-walled cylindrical shells with free-surface effects remain relatively limited, and 3) benchmark analytical results suitable for validating numerical simulations are scarce.

To address these limitations, the present study develops a Rayleigh–Ritz-based analytical formulation combined with Lagrange's equations to evaluate the free vibration characteristics of partially filled cylindrical shells. The proposed model incorporates fluid effects through an analytical added-mass representation derived from velocity potential theory. The influence of liquid filling ratio and shell aspect ratio on the first and second vibration modes is systematically investigated, providing practical benchmark solutions for engineering applications and future numerical studies.

3 | Research Methodology

The present study develops a semi-analytical framework to investigate the free vibration characteristics of a partially liquid-filled thin-walled cylindrical shell. The methodology combines the Rayleigh–Ritz approximation technique with Lagrange's equations to derive the governing equations of the coupled fluid–structure system.

A vertical circular cylindrical shell with uniform thickness is considered. The shell is assumed to be linearly elastic, homogeneous, and isotropic, characterized by Young's modulus, Poisson's ratio, density, radius, height, and thickness. The lower edge of the shell is simply supported and fixed to a rigid foundation, whereas the upper edge is free.

The shell contains a liquid with a prescribed filling height. The contained fluid is assumed to be incompressible, inviscid, and Newtonian. Since the fluid motion is considered irrotational, the fluid velocity field can be represented using a scalar velocity potential satisfying Laplace's equation.

The displacement field of the shell is expressed in terms of admissible functions satisfying the geometric boundary conditions. The axial, circumferential, and radial displacements are approximated using the Rayleigh–Ritz method, where generalized coordinates are introduced to describe the temporal variation of structural deformation.

The governing equations are obtained through the following procedure:

I. The kinetic energy of the cylindrical shell is formulated based on the assumed displacement field.

II. The strain energy of the shell is derived using the classical thin-shell formulation proposed by Bleich and DiMaggio.

III. The velocity potential of the liquid is obtained by solving Laplace's equation subject to the following boundary conditions:

   - *zero normal velocity at the tank bottom,*
   - *compatibility between fluid radial velocity and shell radial displacement at the fluid–shell interface,*
   - *zero hydrodynamic pressure at the free liquid surface.*

IV. The kinetic energy of the liquid is subsequently determined from the velocity potential formulation.

V. The hydrodynamic influence of the fluid is incorporated into the structural equations through the concept of added mass. The resulting added-mass matrix is assembled and combined with the structural mass matrix.

VI. Lagrange's equations are employed to derive the coupled equations of motion of the fluid–shell system.

VII. Neglecting damping effects, the equations of motion are transformed into a generalized eigenvalue problem of the form:

$$[M]\{\ddot{q}\} + [K]\{q\} = 0,$$

where [M] and [K] denote the global mass and stiffness matrices of the coupled system, respectively.

VIII. Assuming harmonic motion, the eigenvalue problem is solved to determine the natural frequencies and corresponding mode shapes of the system.

Finally, a parametric study is conducted to evaluate the effects of the liquid filling ratio (H/L) and shell aspect ratio (L/R) on the first and second natural frequencies. The obtained results are compared under different geometric configurations to investigate the dynamic behavior of partially filled steel and concrete storage tanks.

3.1 | Analysis of the Method

The investigated system, which is a fluid-containing tank, is shown in *Fig. (1)*. The tank is a vertical circular cylindrical shell with uniform thickness h, radius R, height L, density ϱ, and Young's modulus E. The tank is filled with a fluid up to height H, having density ϱ_l. The fluid in the system is assumed to be incompressible and Newtonian.

In the present work, a cylindrical coordinate system (r,θ,z) is used to define an arbitrary point in the tank–fluid assembly. The displacements in the axial, tangential, and radial directions are denoted by w, v, and u, respectively.

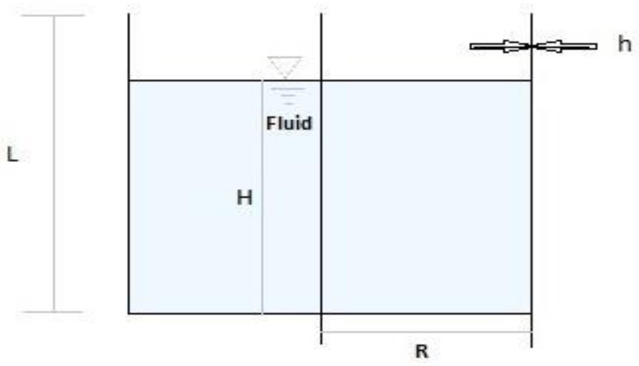

Fig. 1. Fluid-containing tank system.

$$u = \sum_{i=1}^{N_1} U_i\,(t)\chi_i(z)\cos\theta, \tag{1}$$

$$v = \sum_{i=1}^{N_2} V_i(t)\psi_i(z)\sin\theta, \tag{2}$$

$$w = \sum_{i=1}^{N_3} W_i(t)\psi_i(z)\sin\theta, \tag{3}$$

where $W_i(t), V_i(t), U_i(t)$ are the time-dependent displacement amplitudes associated with the functions $\chi_i(z)$, $\psi_i(z)$, and N_3, N_2, N_1 represent the number of these functions used in the displacement approximation.

The governing differential equations are obtained using Lagrange's equation as follows.

$$\frac{d}{dt}\left(\frac{\partial(T_s + T_l)}{\partial \dot{q}_i}\right) - \frac{\partial(T_s + T_l)}{\partial q_i} + \frac{\partial S}{\partial q_i} = 0, \tag{4}$$

where T_s is the kinetic energy of the tank, T_l is the kinetic energy of the fluid, $S$is the strain energy of the tank wall, and q_i are the generalized coordinates.

The kinetic energy of the tank is obtained as follows.

$$T_s = \frac{\rho h}{2}\int_0^{2\pi}\int_0^{L}(\dot{u}^2 + \dot{v}^2 + \dot{w}^2)Rdzd\theta. \tag{5}$$

The dot "·" in the above relations denotes differentiation with respect to time.

The strain energy of the tank is obtained using the formulation given by Bleich and DiMaggio [2] as follows.

$$S = \frac{E}{2(1-\nu^2)}\frac{h}{R}\int_0^{2\pi}\int_0^{L}[R^2\,{u_z}^2 + (v_\theta + w)^2 \tag{6}$$

$$+2\nu R u_z(v_\theta + w) + \frac{1-\nu}{2}(u_\theta + R v_z)^2]dzd\theta$$

$$+ \frac{E}{24(1-\nu^2)}\frac{h^3}{R^3}\int_0^{2\pi}\int_0^H [R^4 {w_{zz}}^2 + (w_{\theta\theta} + w)^2$$

$$+\frac{1-\nu}{2}(R w_{z\theta} - u_\theta)^2 + \frac{3(1-\nu)}{2} R^2(v_z + w_{z\theta})^2$$

$$+2\nu R^2 w_{zz}(w_{\theta\theta} - v_\theta) - 2R^3 u_z w_{zz}]dzd\theta.$$

The expressions within the first and second brackets represent the tensile and bending energies, respectively.

By substituting the expressions of S and T_s into *Eq. (4)*, and eliminating T_l, a set of governing equations for the free vibration analysis of empty tanks is obtained. The resulting equations can be expressed in the following matrix form [3]:

$$\begin{bmatrix} [A] & 0 & 0 \\ 0 & [B] & 0 \\ 0 & 0 & [C] \end{bmatrix}\begin{Bmatrix} \ddot{U} \\ \ddot{V} \\ \ddot{W} \end{Bmatrix} + \begin{bmatrix} [D] & [E] & [F] \\ [E] & [G] & [H] \\ [F] & [H] & [I] \end{bmatrix}\begin{Bmatrix} \{U\} \\ \{V\} \\ \{W\} \end{Bmatrix} = 0. \quad \textbf{(7)}$$

All of the above matrices are square matrices, and $\{W\}, \{V\}, \{U\}$ are column sub-vectors. $U_i = 1,2,\dots,N_1$, $V_i = 1,2,\dots,N_2$, and $W_i = 1,2,\dots,N_3$ represent the components of the vectors $\{U\}, \{V\}, \{W\}$, respectively. The elements of matrices $[A]$ to $[I]$ and the details of the derivation of *Eq. (7)* can be found in Tang and Chang's work [3].

In the absence of damping, the compact form of *Eq. (7)* can be written as follows.

$$[M]\{\ddot{q}\} + [K]\{q\} = \{0\}, \quad \textbf{(8)}$$

where $[M]$ is the mass matrix and $[K]$ is the stiffness matrix. The column vector $\{q\}$ is defined as follows.

$$\{q\} = \begin{Bmatrix} U_1(t), U_2(t), \dots U_{N_1}(t) \\ , V_1(t), V_2(t), \dots V_{N_2}(t) \\ , W_1(t), W_2(t), \dots W_{N_3}(t) \end{Bmatrix}^T. \quad \textbf{(9)}$$

The superscript T denotes matrix transposition.

The governing equations for a fluid-filled shell are obtained by adding the added mass of the fluid to the mass matrix of the empty tank. The added fluid mass is only associated with radial displacement, and the presence of the fluid has no effect on the stiffness matrix $[K]$.

If c_{ij} are the elements of the submatrix $[C]$ in *Eqs. (3-7)*, then c_{ij}^* is defined as

$$c_{ij}^* = c_{ij} + m_{ij}^l, \quad \textbf{(10)}$$

where m_{ij}^l is referred to as the added fluid mass.

The fluid is assumed to be inviscid and incompressible, and its velocity potential satisfies Laplace's equation.

$$\nabla^2\Phi = 0 0 \le r \le R,\ 0 \le \theta \le 2\pi,\ 0 \le z \le H. \quad \textbf{(11)}$$

The fluid velocity component in the axial (z) direction at an arbitrary point can be expressed in terms of the scalar potential Φ as

$$v_z = \frac{\partial\Phi}{\partial z}. \quad \textbf{(12)}$$

The boundary conditions for the fluid field are as follows.

I. The vertical velocity of the fluid at the tank base is zero

$$\frac{\partial \Phi}{\partial z}\Big|_{z=0} = 0. \tag{13}$$

II. The radial velocity of the fluid at the tank wall is equal to the wall velocity:

$$-\frac{\partial \Phi}{\partial r}\Big|_{r=R} = \dot{w} = \sum_{i=1}^{N_3} \dot{W}_i(t)\psi_i(z)\cos\theta. \tag{14}$$

III. At the free surface, the pressure is zero:

$$\frac{\partial \Phi}{\partial t}\Big|_{z=H} = 0. \tag{15}$$

By solving Laplace's equation using the method of separation of variables and applying boundary *Conditions (13)* and *(15)*, the velocity potential $\Phi(r, \theta, z, t)$ is obtained as

$$\Phi(r,\theta,z,t) = \sum_{n=1}^{\infty} C_n(t) I_1\left(\frac{\alpha_n r}{H}\right)\cos\left(\frac{\alpha_n z}{H}\right)\cos\theta, \tag{16}$$

where I_1 is the modified Bessel function of the first kind, $C_n(t)$ is a time-dependent function determined from boundary *Condition (14)*, and $\alpha_n = \frac{(2n-1)\pi}{2}$. After determining $C_n(t)$ from *Eq. (14)* and substituting into *Eq. (16)*, the potential function becomes:

$$\Phi = -\sum_{i=1}^{N_3} \dot{w}_i(t) \sum_{n=1}^{\infty} \frac{2H}{\alpha_n} \frac{I_1\left(\frac{\alpha_n r}{H}\right)}{I_1'\left(\frac{\alpha_n R}{H}\right)} d_{in} \cos\left(\frac{\alpha_n z}{H}\right), \tag{17}$$

where

$$d_{in} = \frac{1}{H}\int_0^H \psi_i(z)\cos\left(\frac{\alpha_n z}{H}\right) d\left(\frac{z}{H}\right). \tag{18}$$

The kinetic energy of an incompressible inviscid fluid is evaluated using Curie's formulation [4] as

$$T_l = \frac{\rho_l}{2}\iint \Phi \frac{\partial \Phi}{\partial n} ds, \tag{19}$$

where Φ is the velocity potential, $\partial\Phi/\partial n$ is the normal derivative of Φ at the boundary, and the surface integral covers the wetted surface of the fluid domain. Only the non-zero contribution of the surface integral corresponds to the tank wall. The contribution at the base is zero due to zero vertical velocity, and the free surface contribution is zero due to the zero-pressure assumption.

After simplification, the fluid kinetic energy becomes:

$$T_l = \frac{\rho_l}{2}\int_0^{2\pi}\int_0^H \left[\Phi\frac{\partial \Phi}{\partial r}\right]_{r=R} R\,dz\,d\theta. \tag{20}$$

Substituting *Eq. (17)* into *Eq. (20)* and performing the required integrations yields:

$$T_l = \frac{1}{2}\sum_{i=1}^{N_3}\sum_{j=1}^{N_3} m_{ij}^l \dot{w}_i \dot{w}_j, \tag{21}$$

where

$$m_{ij}^{l} = \left[\frac{H}{R}\sum_{n=1}^{\infty}\frac{2}{\alpha_n}\frac{I_1(\alpha_n R/H)}{I_1'(\alpha_n R/H)}d_{in}d_{jn}\right]m_l, \tag{22}$$

and $m_l = \pi\rho_l R^2 H$ is the total fluid mass. *Eq. (22)* represents the added mass of the fluid used in *Eq. (10)*.

By substituting the mass matrix coefficients C_{ij} into the mass matrix [M], and assuming harmonic time dependence, *Eq. (8)* reduces to the standard eigenvalue problem:

$$[K]\{q\} = \omega^2[M]\{q\}, \tag{23}$$

where {q} is the eigenvector representing the displacement amplitudes, and $\omega$is the circular natural frequency of the shell–fluid system.

4 | Numerical Solution

A flexible steel tank with Young's modulus $E = 30 \times 10^3$ psi, density $\rho = 490$ lb, height $L = 92$ in, and thickness $h = 0.052$ in is considered, containing a fluid with density $\rho_l = 62.4$ lb/in³ and various fluid heights. The natural frequencies of the tank for different values of H/L and L/R ratios for the first and second modes are presented in the following tables.

Table 1. Natural frequency of the steel tank (first mode).

$\frac{h}{R} = 0.001,\ \nu = 0.3,\ \frac{\rho_l}{\rho} = 0.127$				
$\frac{L}{R}$	$\frac{H}{L} = 1$	$\frac{H}{L} = 0.8$	$\frac{H}{L} = 0.6$	$\frac{H}{L} = 0.5$
0.3	6.3964	7.9944	10.6579	12.7889
0.5	6.4012	7.9981	10.6607	12.7913
0.75	6.4107	8.0057	10.6663	12.7959
1	6.4245	8.0164	10.6742	12.8024
1.5	6.4670	8.0487	10.6974	12.8214
2	6.5333	8.0977	10.7316	12.8490
2.5	6.6279	8.1666	10.7783	12.8862
3	6.7524	8.2581	10.8392	12.9339

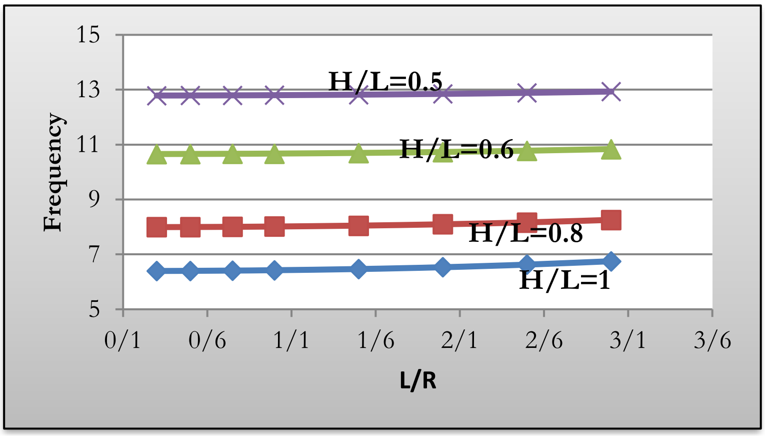

Fig. 2. Natural frequency of the steel tank (first mode).

Table 2. Natural frequency of the steel tank (second mode).

$\frac{h}{R}=0.001,\ \nu=0.3,\ \frac{\rho_l}{\rho}=0.127$				
$\frac{L}{R}$	$\frac{H}{L}=1$	$\frac{H}{L}=0.8$	$\frac{H}{L}=0.6$	$\frac{H}{L}=0.5$
0.3	6.4045	8.0008	10.6627	12.7929
0.5	6.4245	8.0164	10.6742	12.8024
0.75	6.4670	8.0487	10.6974	12.8214
1	6.5333	8.0977	10.7316	12.8490
1.5	6.7525	8.2582	10.8393	12.9339
2	7.0786	8.5129	11.0109	13.0665
2.5	7.4677	8.8483	11.2542	13.2557
3	7.8808	9.2335	11.5629	13.5048

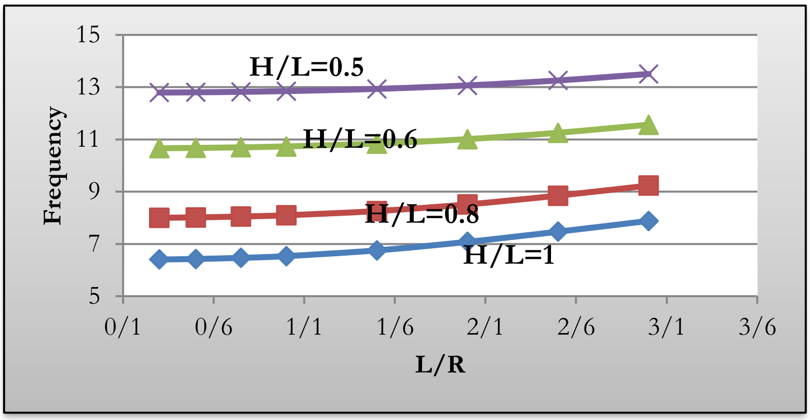

Fig. 3. Natural frequency of the steel tank (second mode).

By keeping the tank height constant L, the above results were obtained for the steel fluid-filled tank in the first and second modes for different ratios of fluid height to tank height $\frac{H}{L}$ as well as tank height to radius $\frac{L}{R}$. These results show that by decreasing the ratio $\frac{H}{L}$ and increasing the ratio radius $\frac{L}{R}$, the natural frequency increases in both the first and second modes. Similar results were also obtained for concrete tanks in the first and second modes, which are presented in the following tables.

Table 3. Natural frequency of the concrete tank (first mode).

$\frac{h}{R}=0.01,\ \nu=0.15,\ \frac{\rho_l}{\rho}=0.5$				
$\frac{L}{R}$	$\frac{H}{L}=1$	$\frac{H}{L}=0.8$	$\frac{H}{L}=0.6$	$\frac{H}{L}=0.5$
0.3	10.3036	12.8767	17.1661	20.5976
0.5	10.3149	12.8857	17.1727	20.6034
0.75	10.3378	12.9035	17.1859	20.6143
1	10.3714	12.9295	17.2047	20.6298
1.5	10.4779	13.0090	17.2609	20.6755
2	10.6480	13.1330	17.3453	20.7428
2.5	10.8890	13.3100	17.4631	20.8350
3	11.1938	13.5439	17.6191	20.9556

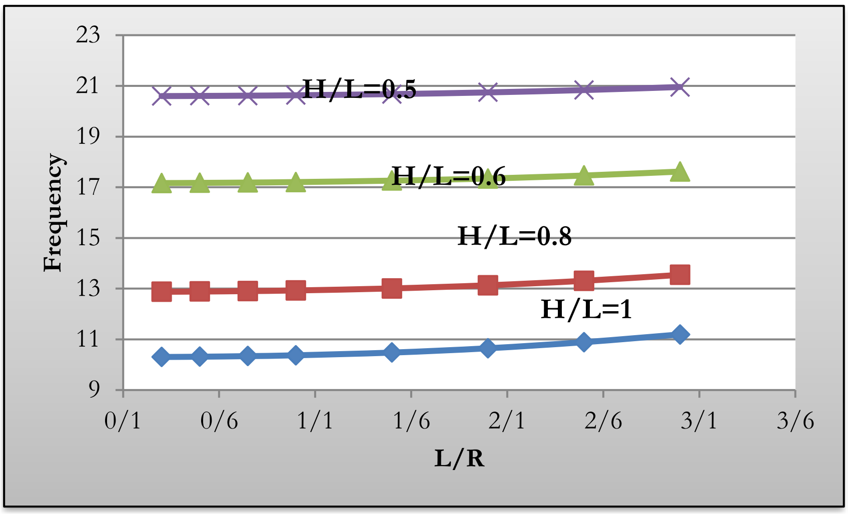

Fig. 4. Natural frequency of the concrete tank (first mode).

Table 4. Natural frequency of the concrete tank (second mode).

$\frac{h}{R}=0.01,\ \nu=0.15,\ \frac{\rho_l}{\rho}=0.5$				
$\frac{L}{R}$	$\frac{H}{L}=1$	$\frac{H}{L}=0.8$	$\frac{H}{L}=0.6$	$\frac{H}{L}=0.5$
0.3	10.3328	12.8919	17.1661	20.6069
0.5	10.3714	12.9295	17.1727	20.6298
0.75	10.4778	13.0090	17.1859	20.6755
1	10.6480	13.1330	17.2047	20.7428
1.5	11.1941	13.5442	17.2609	20.9557
2	11.9275	14.1627	17.3453	21.2960
2.5	12.7290	14.9094	17.4631	21.7780
3	13.5364	15.7082	17.6191	22.3875

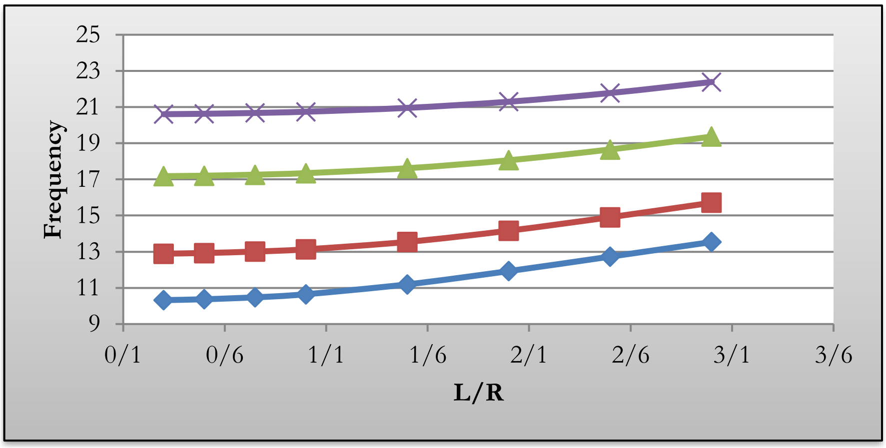

Fig. 5. Natural frequency of the concrete tank (second mode).

5 | Conclusion

In this section, based on the performed analyses and obtained responses, several conclusions of this study are presented:

I. By increasing the tank height-to-radius ratio and decreasing the fluid height-to-tank height ratio, the natural frequency of the fluid-filled tank system increases for both steel and concrete tanks in the first and second modes. For instance, for a steel fluid-filled tank in the first mode with $L/R = 0.3$, the natural frequencies corresponding to $H/L = 1{,}0.8{,}0.6{,}0.5$ are 6.3964, 7.9944, 10.6579, and 12.7889, respectively. In the second mode, the corresponding values are 6.4045, 8.008, 10.6627, and 12.7929. For $L/R = 3$ and the same H/L ratios, the natural frequencies in the first mode are 6.7524, 8.2581, 10.8392, and 12.9339, respectively, while in the second mode they are 7.8808, 9.2335, 11.5629, and 13.5048, respectively.

II. For the concrete fluid-filled tank, the natural frequency in the first mode for $L/R = 0.3$ and $H/L = 1{,}0.8{,}0.6{,}0.5$ is 10.3036, 12.8767, 17.1661, and 20.5976, respectively, while in the second mode the corresponding values are 10.3228, 12.8919, 17.1774, and 20.6069. For $L/R = 3$ and the same H/L ratios, the natural frequencies in the first mode are 11.1938, 13.5439, 17.6191, and 20.9556, respectively, while in the second mode they are 13.5364, 15.7082, 19.3670, and 22.3875, respectively

References

[1] Bleich, H. H., & DiMaggio, F. (2021). A strain-energy expression for thin cylindrical shells. *Journal of applied mechanics*, *20*(3), 448–449. https://doi.org/10.1115/1.4010739

[2] Yang, J. Y. (1976). *Dynamic behavior of fluid tank systems* [Thesis].

[3] Tang, Y., & Chang, Y. W. (1994). *Free vibration analysis of partially filled liquid storage tanks*. https://www.osti.gov/biblio/10154429

[4] Currie, I. G. (2012). *Fundamental mechanics of fluids, fourth edition*. Taylor & Francis. https://books.google.com/books?id=3nOXMcEqL38C

[5] Amabili, M. (2008). *Nonlinear vibrations and stability of shells and plates*. Cambridge University Press. https://doi.org/10.1017/CBO9780511619694

[6] Amabili, M., & Paı¨doussis, M. P. (2003). Review of studies on geometrically nonlinear vibrations and dynamics of circular cylindrical shells and panels, with and without fluid-structure interaction. *Applied mechanics reviews*, *56*(4), 349–381. https://doi.org/10.1115/1.1565084

[7] Ricciardi, G. (2016). Fluid–structure interaction modelling of a PWR fuel assembly subjected to axial flow. *Journal of fluids and structures*, *62*, 156–171. https://doi.org/10.1016/j.jfluidstructs.2016.01.006

[8] Païdoussis, M. P., Price, S. J., & de Langre, E. (2014). *Fluid-structure interactions: Cross-flow-induced instabilities*. Cambridge University Press. https://books.google.com/books?id=0cUvnwEACAAJ

[9] Chen, S., Han, Y., Zhu, X., Li, T., & Zhong, K. (2024). Coupled vibration analysis of partially liquid-filled cylindrical shells with internal horizontal plates. *Thin-walled structures*, *200*, 111884. https://doi.org/10.1016/j.tws.2024.111884

[10] Montes, R. O. P., Silva, F. M. A., & Pedroso, L. J. (2024). Free vibration analysis of a clamped cylindrical shell with internal and external fluid interaction. *Journal of fluids and structures*, *125*, 104079. https://doi.org/10.1016/j.jfluidstructs.2024.104079

[11] Montes, R. O. P., da Silva, F. M. A., & Pedroso, L. J. (2025). A semi-analytical model for nonlinear vibration analysis of clamped, submerged, and fluid-filled cylindrical shells. *Journal of the brazilian society of mechanical sciences and engineering*, *47*(2), 102. https://doi.org/10.1007/s40430-025-05385-4

[12] Fang, X., Bao, X., Yue, F., & Zhao, Q. (2024). A dynamic analysis method of liquid-filled containers considering the fluid–structure interaction. *Applied sciences*, *14*(7), 2688. https://doi.org/10.3390/app14072688

[13] Saboo, A., & Kumar, M. (2023). Dynamic analysis of a partially-filled cylindrical–conical–cylindrical shell representing a pressure vessel. *Thin-walled structures*, *183*, 110342. https://doi.org/10.1016/j.tws.2022.110342

[14] Zhao, T., Ye, T., Chen, Y., Jin, G., & Liu, Z. (2024). Vibroacoustic analysis of submerged fluid-filled cylindrical shell. *International journal of mechanical sciences*, *275*, 109330. https://doi.org/10.1016/j.ijmecsci.2024.109330

[15] Sun, Y., Meng, X., Zhang, Z., Gu, Z., Wang, J., & Zhou, D. (2024). Liquid sloshing in soil-supported multiple cylindrical tanks equipped with baffle under horizontal excitation. *Buildings*, *14*(4), 1029. https://doi.org/10.3390/buildings14041029